

CFT Duals of Black Rings With Higher Derivative Terms

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Abstract

We study possible CFT duals of supersymmetric five dimensional black rings in the presence of supersymmetric higher derivative corrections to the $\mathcal{N} = 2$ supergravity action. A Virasoro algebra associated to an asymptotic symmetry group of solutions is defined by using the Kerr/CFT approach. We find the central charge and compute the microscopic entropy which is in precise agreement with the macroscopic entropy. Although apparently related to a different aspect of the near-horizon geometry and a different Virasoro algebra, we find that the c-extremization method leads to the same central charge and microscopic entropy computed in the Kerr/CFT approach. The relationship between these two point of view is clarified by relating the geometry to a self-dual orbifold of AdS_3 .

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1 Introduction

A decade before Maldacena's AdS/CFT conjecture of gravity-gauge duality [1], a relationship between gravity on AdS_3 and the two dimensional conformal group was found by Brown and Henneaux [2]. They found that appropriate boundary conditions, which preserve the asymptotically AdS_3 geometry, generate two Virasoro algebras. It is not a great leap to relate this asymptotic symmetry group to some CFT living on the boundary and applying the Brown-Henneaux formalism to asymptotically AdS black hole solutions allows one to relate the macroscopic Bekenstein-Hawking entropy to the number of states of a boundary CFT [3, 4, 5, 6]. One uses the fact that if the asymptotic symmetry group is a Virasoro algebra with a particular central charge, the Cardy formula relates the central charge to entropy of the CFT in the large charge limit. We refer to the entropy obtained from the Brown-Henneaux formalism as the microscopic entropy but it should be recalled that, in many cases, the existence of a microscopic CFT is only hypothesised.

More recently, a clever extension of the Brown-Henneaux approach to the $\text{SL}(2, \mathbb{R}) \times U(1)$ near-horizon symmetry of the extremal Kerr black hole, led to the proposal of a Kerr/CFT correspondence [7]. It was found that one can find an asymptotic symmetry group, corresponding to the $U(1)$ part of the isometry, with a Virasoro algebra whose central charge correctly accounts for the Bekenstein-Hawking entropy via the Cardy formula.

The Kerr/CFT correspondence has been generalised to, and verified for higher dimensional rotating black holes [8, 9, 10, 11, 12, 13, 14, 15], Lagrangians with topological terms in four and five dimensions [16] and Lagrangians with higher derivative terms [17]. Notably, it was shown that in certain theories with gravity coupled to matter fields the associated central charge coming from the gravitational degrees of freedom is sufficient to account for the entropy [11, 16].

Recently, people have tried to embed the Kerr/CFT approach in string theory [18, 19, 20]. In light of this we were motivated to study possible CFT duals of five dimensional supersymmetric black rings [21] in the presence of special higher derivative terms which are the supersymmetric completion of a mixed gauge-gravitational Chern-Simons term [22]. These solutions have $\text{SL}(2, \mathbb{R}) \times U(1) \times \text{SO}(3)$ near-horizon symmetries corresponding to an S^1 fibred over $\text{AdS}_2 \times S^2$. For various technical reasons outlined below, this seems to be a promising case to study. Firstly, with the addition of higher derivative terms one might

hope to study corrections to the entropy beyond the Cardy limit. It turns out that the near horizon geometry retains the same form upon addition of these higher derivative terms [23, 24, 25], making calculations tractable. Secondly, the rich near horizon geometry of the SUSY black rings allows for various approaches to finding the entropy to be studied and compared. In particular, the S^1 part, which carries the angular momentum of the black ring, is fibred over the AdS_2 part such that locally one has an AdS_3 geometry called a self-dual orbifold of AdS_3 [26, 27]. So in addition to using a generalisation of Kerr/CFT, corresponded to the $U(1)$ part of the isometry of near horizon geometry [8], the fact that one locally has an AdS_3 means that the c-extremisation formalism [28] can be applied. This approach is based on the relationship between the conformal anomaly and the variation of the gravitational action with respect to a metric of the form $\text{AdS}_3 \times Y$ where Y is compact. One obtains two Virasoro algebras corresponded to the $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ isometry of AdS_3 .

We found that comparing the result of applying Kerr/CFT and c-extremisation illuminating. The fact that one only has a local AdS_3 means an application of the c-extremisation formalism is not straight forward. The $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ isometry of AdS_3 is reduced to $SL(2, \mathbb{R})_L \times U(1)$. On the CFT side one expects that this kills the right-handed excitations, which means there is a (0,4)-CFT corresponded to SUSY black ring, and the microscopic entropy is given by one part of the Cardy formula, [29, 30, 31, 32]

$$S_{\text{mic}} = \sqrt{\frac{c_L \hat{q}_0}{6}}. \quad (1.1)$$

On the other hand, in the Kerr/CFT approach, where the Virasoro is intimately related to the $U(1)$ part of the isometry, which descends from the right-handed $SL(2, \mathbb{R})_R$, one may naively expect that the central charge obtained descends from the right-handed central charge of global AdS_3 . This is not the case. From CFT point of view, we know that there is a (0,4)-CFT corresponding to the SUSY black ring [29, 30, 31, 32] and we expect that the left central charge contributes to the microscopic entropy. We will show that both c-extremization and Kerr/CFT approaches lead to the same central charge which is the left central charge c_L . This agreement was shown for supersymmetric black ring without higher derivative terms [8]. The equality of the two central charges when higher derivative corrections are added is a non-trivial new result.

The rest of this paper is organised as follows. In section 2 we briefly review supersymmetric black ring solutions of five dimensional superconformal gravity in the presence of higher derivative terms. Then we apply the Kerr/CFT approach to the supersymmetric black ring in section 3. The associated central charge is computed and the agreement between the microscopic entropy and macroscopic entropy is shown. Section 4 is devoted to the application of the c-extremization formalism for the above black ring solution. We show that the associated left central charge and microscopic entropy are in agreement with the Kerr/CFT results. Finally in section 5, we will summarize and discuss our results.

2 5D Supergravity with higher derivative terms

In this section we review five dimensional $\mathcal{N} = 2$ supergravity with higher derivative corrections associated with a mixed gauge gravitational Chern-Simons term. We will do that in the context of an off-shell formalism which involves superconformal gravity [22]. The

important feature of the formalism is that variation of fermionic fields does not depend explicitly on the form of the action. In particular, the variation of fermionic fields does not change if higher order derivative terms are added.

Compactification of M-theory on a six dimensional Calabi-Yau manifold results in $\mathcal{N} = 2$ supergravity in five dimensions. In [22] it was shown that by enlarging the symmetries to superconformal gravity (by adding some auxiliary fields) one can find a generic form for the fermionic variations which leave the action invariant. As mentioned, this form is valid for any number of higher derivative terms. The bosonic action, up to 4th order, is given by³

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{|g|} (\mathcal{L}_0 + \mathcal{L}_1), \quad (2.1)$$

where

$$\begin{aligned} \mathcal{L}_0 = & \partial_a \mathcal{A}_\alpha^i \partial^a \mathcal{A}_i^\alpha + (2\nu + \mathcal{A}^2) \frac{D}{4} + (2\nu - 3\mathcal{A}^2) \frac{R}{8} + (6\nu - \mathcal{A}^2) \frac{v^2}{2} + 2\nu_I F_{ab}^I v^{ab} \\ & + \frac{1}{4} \nu_{IJ} (F_{ab}^I F^{Jab} + 2\partial_a X^I \partial^a X^J) + \frac{e^{-1}}{24} C_{IJK} \epsilon^{abcde} A_a^I F_{bc}^J F_{de}^K, \end{aligned} \quad (2.2)$$

is the tree level part of the action and

$$\begin{aligned} \mathcal{L}_1 = & \frac{c_{2I}}{24} \left(\frac{1}{16e} \epsilon_{abcde} A^{Ia} R^{bcfg} R_{fg}^{de} + \frac{1}{8} X^I C^{abcd} C_{abcd} + \frac{1}{12} X^I D^2 + \frac{1}{6} F^{Iab} v_{ab} D \right. \\ & + \frac{1}{3} X^I C_{abcd} v^{ab} v^{cd} + \frac{1}{2} F^{Iab} C_{abcd} v^{cd} + \frac{8}{3} X^I v_{ab} \hat{D}^b \hat{D}_c v^{ac} \\ & + \frac{4}{3} X^I \hat{D}^a v^{bc} \hat{D}_a v_{bc} + \frac{4}{3} X^I \hat{D}^a v^{bc} \hat{D}_b v_{ca} - \frac{2}{3e} X^I \epsilon_{abcde} v^{ab} v^{cd} \hat{D}_f v^{ef} \\ & + \frac{2}{3e} F^{Iab} \epsilon_{abcde} v^{cf} \hat{D}_f v^{de} + e^{-1} F^{Iab} \epsilon_{abcde} v^c{}_f \hat{D}^d v^{ef} \\ & \left. - \frac{4}{3} F^{Iab} v_{ac} v^{cd} v_{db} - \frac{1}{3} F^{Iab} v_{ab} v^2 + 4X^I v_{ab} v^{bc} v_{cd} v^{da} - X^I (v^2)^2 \right), \end{aligned} \quad (2.3)$$

are all four derivative terms which are related to the mixed gauge-gravitational Chern-Simons term $c_{2I} A^I \wedge R \wedge R$ by supersymmetry transformations [22]. In this action C_{IJK} and c_{2I} are the intersection numbers and the second Chern class of internal space CY_3 respectively, $\mathcal{A}^2 = \mathcal{A}_\alpha^i \mathcal{A}_i^\alpha$, $v^2 = v_{ab} v^{ab}$ and

$$\nu = \frac{1}{6} C_{IJK} X^I X^J X^K, \quad \nu_I = \frac{1}{2} C_{IJK} X^J X^K, \quad \nu_{IJ} = C_{IJK} X^K. \quad (2.4)$$

The fields appearing in the action are arranged in Weyl, vector and hyper multiplets. The Weyl multiplet contains the metric, a 2-form auxiliary field, v_{ab} , a scalar auxiliary field D , a gravitino ψ_μ^i and an auxiliary Majorana spinor χ^i . Each vector multiplet contains a 1-form gauge field A^I , a scalar auxiliary field X^I and a gaugino Ω^{Ii} (where $I = 1, \dots, n_v$ count the number of vector multiplets) and $i = 1, 2$ is an $SU(2)$ doublet index and $\alpha = 1, \dots, 2r$ refers to $USP(2r)$ group. The hyper multiplet contains the auxiliary scalar fields \mathcal{A}_α^i and a hyperino ζ^α .

³We will use units $G_5 = \pi/4$ for the five dimensional Newton's constant.

The bosonic terms of supersymmetry variation of fermions are ⁴

$$\begin{aligned}
\delta\psi_\mu^i &= \mathcal{D}_\mu \varepsilon^i + \frac{1}{2} v^{ab} \gamma_{\mu ab} \varepsilon^i - \gamma_\mu \eta^i, \\
\delta\chi^i &= D\varepsilon^i - 2\gamma^c \gamma^{ab} \hat{\mathcal{D}}_a v_{bc} \varepsilon^i + \gamma^{ab} \hat{R}_{ab}(V)_j^i \varepsilon^i - 2\gamma^a \varepsilon^i \epsilon_{abcde} v^{bc} v^{de} + 4\gamma^{ab} v_{ab} \eta^i, \\
\delta\Omega^{Ii} &= -\frac{1}{4} \gamma^{ab} F_{ab}^I \varepsilon^i - \frac{1}{2} \gamma^a \partial_a X^I \varepsilon^i - X^I \eta^i, \\
\delta\zeta^\alpha &= \gamma^a \partial_a \mathcal{A}_i^\alpha - \gamma^{ab} v_{ab} \varepsilon^i \mathcal{A}_i^\alpha + 3\mathcal{A}_i^\alpha \eta^i,
\end{aligned} \tag{2.5}$$

where $\delta \equiv \bar{\epsilon}^i \mathbf{Q}_i + \bar{\eta}^i \mathbf{S}_i + \xi_K^a \mathbf{K}_a$ ⁵ and the covariant derivatives are defined by

$$\mathcal{D}_\mu \varepsilon^i = \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} + \frac{1}{2} b_\mu \right) \varepsilon^i - V_\mu^i{}_j \varepsilon^j, \tag{2.6}$$

$$\hat{\mathcal{D}}_\mu v_{ab} = (\mathcal{D}_\mu - b_\mu) v_{ab} = \partial_\mu v_{ab} + 2\omega_{[\mu}^c v_{b]c} - b_\mu v_{ab}, \tag{2.7}$$

in which b_μ is a real boson in the Weyl multiplet and is $SU(2)$ singlet [22].

There is a well-known gauge to fix the conformal invariance of the off-shell formalism and reduce the superconformal symmetry to the standard symmetries of five dimensional $\mathcal{N} = 2$ supergravity,

$$\mathcal{A}^2 = -2, \quad b_\mu = 0, \quad V_\mu^{ij} = 0. \tag{2.8}$$

In this gauge the last equation of (2.5) gives η^i in terms of ε^i as,

$$\eta^i = \frac{1}{3} \gamma^{ab} v_{ab} \varepsilon^i. \tag{2.9}$$

In the gauge (2.8) and using above equation (2.9) the supersymmetry variations (2.5) simplify to

$$\begin{aligned}
\delta\psi_\mu^i &= \left(\mathcal{D}_\mu + \frac{1}{2} v^{ab} \gamma_{\mu ab} - \frac{1}{3} \gamma_\mu \gamma^{ab} v_{ab} \right) \varepsilon^i, \\
\delta\chi^i &= \left(D - 2\gamma^c \gamma^{ab} \mathcal{D}_a v_{bc} - 2\gamma^a \epsilon_{abcde} v^{bc} v^{de} + \frac{4}{3} (\gamma^{ab} v_{ab})^2 \right) \varepsilon^i, \\
\delta\Omega^{Ii} &= \left(-\frac{1}{4} \gamma^{ab} F_{ab}^I - \frac{1}{2} \gamma^a \partial_a X^I - \frac{1}{3} X^I \gamma^{ab} v_{ab} \right) \varepsilon^i.
\end{aligned} \tag{2.10}$$

In the next subsection we review the supersymmetric black ring solution of $\mathcal{N} = 2$ five dimensional supergravity in the presence of higher derivative supersymmetric corrections (2.3).

2.1 Black Ring solution

To compute the entropy of an extremal black hole we just need to know the near horizon geometry. In [25] the near horizon of five dimensional supersymmetric black ring in the presence of higher derivative terms (2.3) is derived using the entropy function formalism [33]. In addition by using the entropy function formalism the macroscopic entropy of black

⁴Here $\gamma_{a_1 a_2 \dots a_m} = \frac{1}{m!} \gamma_{[a_1} \gamma_{a_2} \dots \gamma_{a_m]}$ which is antisymmetric in all indices. Also the covariant curvature $\hat{R}_{\mu\nu}^{ij}$ is defined by $\hat{R}_{\mu\nu}^{ij} = 2\partial_{[\mu} V_{\nu]}^{ij} - 2V_{[\mu}^i V_{\nu]}^{kj} + \text{fermionic terms}$, where V_μ^{ij} is a boson in the Weyl multiplet which is in $\mathbf{3}$ of the $SU(2)$. For the solution we are going to consider, this term vanishes.

⁵ \mathbf{Q}_i is the generator of $\mathcal{N} = 2$ supersymmetry, \mathbf{S}_i is the generator of conformal supersymmetry and \mathbf{K}_a are special conformal boost generators of superconformal algebra [22].

ring is computed. Unfortunately it is non-trivial to extract an expression for the entropy purely as a function of the physical charges using this formalism [23].

The near horizon geometry of black ring solutions with higher derivative terms (2.3) can be also found directly by solving the near horizon equations of motion [34]. In this subsection we report the result of these calculations and discuss the symmetries of the geometry in detail.

The near horizon of supersymmetric black ring in five dimensions is given by

$$\begin{aligned} ds^2 &= l_{AdS^2}^2 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + l_{S^1}^2 (d\psi + e_0 r dt)^2 + l_{S^2}^2 (d\theta^2 + \sin^2 \theta d\phi^2), \\ A^I &= e^I r dt - \frac{p^I}{2} \cos \theta d\phi + a^I (e_0 r dt + d\psi), \quad X^I = \frac{p^I}{l_{AdS^2}}, \quad D = \frac{12}{l_{AdS^2}^2}, \\ Q^I &= -4C_{IJK} p^J a^K, \quad e^I + e_0 a^I = 0, \quad v_{\theta\phi} = \frac{3}{8} l_{AdS^2} \sin \theta, \end{aligned} \quad (2.11)$$

in which θ and ϕ are the coordinates of a usual 2-sphere and ψ is the coordinate of ring and is periodic, $\psi \sim \psi + 4\pi$, Q^I are the electric charges and the radii are given by the magnetic charges p^I ,

$$l_{AdS_2} = l_{S^2} = e_0 l_{S^1} = \frac{1}{2} \left(\frac{1}{6} C_{IJK} p^I p^J p^K + \frac{1}{12} c_{2I} p^I \right)^{1/3}. \quad (2.12)$$

From (2.11) we can see that the metric consists of a $U(1)$ fibred over an AdS_2 base times a two-sphere. In other words the isometries of the metric are $SL(2, \mathbb{R}) \times U(1) \times SO(3)$ and are generated by

$$\begin{aligned} L_0 &= r\partial_r - t\partial_t, \quad L_1 = (t^2 + r^{-2})\partial_t - 2rt\partial_r - \frac{2e_0}{r}\partial_\psi, \quad L_{-1} = \partial_t, \\ \bar{L}_0 &= -e_0\partial_\psi, \\ J^\pm &= -i\partial_\phi, \quad J^\pm = e^{\pm i\phi}(-i\partial_\theta \pm \cot \theta \partial_\phi). \end{aligned} \quad (2.13)$$

In fact we can think of the first part of the metric as locally AdS_3 with the symmetries $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ with the one of the $SL(2, \mathbb{R})$'s broken to a $U(1)$. One can show that we also have the locally defined killing vectors

$$\bar{L}_1 = e^{\frac{\psi}{e_0}} \left(\frac{1}{r}\partial_t + r\partial_r - e_0\partial_\psi \right), \quad \bar{L}_{-1} = e^{-\frac{\psi}{e_0}} \left(\frac{1}{r}\partial_t - r\partial_r - e_0\partial_\psi \right). \quad (2.14)$$

Notice that since ψ is periodic, $\bar{L}_{\pm 1}$ are not well defined globally. One finds that together with the generator of the $U(1)$ part of the near horizon isometry, \bar{L}_0 , we obtain an $SL(2, \mathbb{R})$ algebra,

$$[\bar{L}_m, \bar{L}_n] = (m - n)\bar{L}_{m+n}, \quad m, n = 0, \pm 1. \quad (2.15)$$

The periodicity of ψ breaks this $SL(2, \mathbb{R})$ to a $U(1)$. This local AdS_3 symmetry will permit us to use the c-extremization approach [28] to find the associated central charge in section 4.

Now, the parameter e_0 gives the angular momentum of the black ring solution in 5D, while if one reduces along the ψ direction, from a 4D point of view, e_0 is an electric field. In the entropy function formalism one can not easily find this electric field in terms of physical charges (or angular velocity) of the black ring [25, 8].

Solving the near horizon equations of motion leads to an additional relation between the parameter e_0 , magnetic charges p^I of the black ring, electric charges Q^I of the black ring and angular velocities J_ϕ and J_ψ ,

$$J_\phi - J_\psi + \frac{1}{8}C^{IJ}(Q_I - C_{IK}p^K)(Q_J - C_{JL}p^L) = \frac{1}{e_0^2} \left(\frac{1}{6}C_{IJK}p^I p^J p^K + \frac{1}{6}c_{2I}p^I \right) \quad (2.16)$$

$$J_\phi = \frac{1}{2}p^I \left(Q_I - \frac{1}{36}C_{IJK}p^J p^K \right), \quad (2.17)$$

where C^{IJ} is the inverse of $C_{IJ} \equiv C_{IJK}p^K$. In [34], it was also shown that the left hand side of eq.(2.16) is equal to the charge \hat{q}_0 , associated with the Kaluza-Klein photon, which will be used to compute the microscopic entropy of black ring in the next section,

$$\hat{q}_0 \equiv J_\phi - J_\psi + \frac{1}{8}C^{IJ}(Q_I - C_{IK}p^K)(Q_J - C_{JL}p^L). \quad (2.18)$$

The macroscopic entropy of supersymmetric five dimensional black ring is given by

$$S_{\text{mac}} = \frac{2\pi}{e_0} \left(\frac{1}{6}C_{IJK}p^I p^J p^K + \frac{1}{6}c_{2I}p^I \right) = 2\pi \sqrt{\frac{\hat{q}_0(C_{IJK}p^I p^J p^K + c_{2I}p^I)}{6}}. \quad (2.19)$$

In the next two sections we will compute the microscopic entropy of black rings with higher derivative corrections (2.3). We will show that both formalisms lead to the same result for the microscopic entropy and the macroscopic entropy calculated in this section (2.19).

3 Kerr/CFT Approach

The microscopic entropy of extremal black rings can be calculated by using the Kerr/CFT approach. This approach can be applied when the near horizon geometry contains a $U(1)$ fibred over AdS_2 which is the case for black rings we consider.

The Kerr/CFT approach was extended to the case with a Chern-Simons term [16]. It was shown that for a theory with gravity and also other fields, the central charge is not affected by non-gravitational fields. This approach was also generalized to theories with higher derivative corrections [17]. Although this generalization was based on four dimensional kerr black hole in the extremal limit we will show that the black ring satisfy the conditions that help us to use the results of [17] to compute the central charge of associated Virasoro algebra in the presence of higher derivative corrections (2.3).

3.1 Asymptotic symmetry group

Since the black ring near horizon geometry with higher derivative corrections is similar to the case without, one can use the same boundary conditions as those used in [8],

$$h_{\mu\nu} \sim \mathcal{O} \begin{pmatrix} r^2 & 1/r^2 & 1/r & r & 1 \\ & 1/r^3 & 1/r^2 & 1/r^2 & 1/r \\ & & 1/r & 1/r & 1/r \\ & & & 1/r & 1 \\ & & & & 1 \end{pmatrix}, \quad (3.1)$$

in the basis $(t, r, \theta, \phi, \psi)$. The generators associated to these boundary conditions are given by

$$\zeta_n = -e^{-in\psi} \partial_\psi - in r e^{-in\psi} \partial_r, \quad (3.2)$$

which satisfy a Virasoro algebra

$$i[\zeta_m, \zeta_n] = (m - n)\zeta_{m+n}. \quad (3.3)$$

Two interesting facts can be noted when comparing (2.13) and (3.2). Firstly, ζ_0 is proportional to \bar{L}_0 which is the generator of the near horizon $U(1)$ symmetry. It is said that the Virasoro is “based” on this $U(1)$. Secondly, the other ζ ’s do not commute with L_1 which is a generator of the near horizon $SL(2, \mathbb{R})$. Furthermore, this non-commutativity is due to the last term of L_1 which is related to the fibration of the $U(1)$ on an AdS_2 base. This means that the Virasoro is not decoupled from the $SL(2, \mathbb{R})$.

To apply the Kerr/CFT approach when higher derivative corrections are added it is useful to do the calculations in a non-basis coordinates. The vielbeins associated to near horizon geometry of black ring are

$$e^{\hat{t}} = l_{AdS_2} r dt, \quad e^{\hat{r}} = \frac{l_{AdS_2}}{r} dr, \quad e^{\hat{\theta}} = l_{AdS_2} d\theta, \quad e^{\hat{\phi}} = l_{AdS_2} \sin \theta d\phi, \quad e^{\hat{\psi}} = l_{S^1} (d\psi + e_0 r dt), \quad (3.4)$$

and the variations of the vielbeins are given by

$$\begin{aligned} \mathcal{L}_{\zeta_n} e^{\hat{t}} &= in e^{-in\psi} e^{\hat{t}}, & \mathcal{L}_{\zeta_n} e^{\hat{r}} &= -e_0 n^2 e^{-in\psi} (e^{\hat{\psi}} - e^{\hat{t}}), \\ \mathcal{L}_{\zeta_n} e^{\hat{\theta}} &= \mathcal{L}_{\zeta_n} e^{\hat{\phi}} = 0, & \mathcal{L}_{\zeta_n} e^{\hat{\psi}} &= in e^{-in\psi} (e^{\hat{\psi}} - 2e^{\hat{t}}). \end{aligned} \quad (3.5)$$

These variations are similar to the case of the Kerr black hole [17].

The Virasoro algebra we found (3.3) corresponds to Poisson brackets between the generators. Since we are interested in studying the quantum behavior of the boundary fluctuations, we need to find the Dirac brackets which may lead to a Virasoro algebra with a central charge. To compute this central charge we follow [35, 36, 37]. The central charge is given by

$$c^{(k)} = 12i \int_{\partial\Sigma} \mathbf{k}_{\zeta_n}^{inv} [\mathcal{L}_{\zeta_n} g; g] \Big|_{n^3} \quad (3.6)$$

where $|_{n^3}$ stands for the term of order n^3 and

$$\mathbf{k}_{\zeta_n}^{inv} [\mathcal{L}_{\zeta_n} g; g] = -2 \left[\mathbf{X}_{cd} \mathcal{L}_{\zeta_n} \nabla^c \zeta_{-n}^d + (\mathcal{L}_{\zeta_n} \mathbf{X})_{cd} \nabla^{[c} \zeta_{-n}^{d]} + \mathcal{L}_{\zeta_n} \mathbf{W}_c \zeta_{-n}^c \right] - \mathbf{E}[\mathcal{L}_{\zeta_n} g, \mathcal{L}_{\zeta_n} g; g], \quad (3.7)$$

in which covariant derivatives are defined with respect to the original metric g . \mathbf{X} and \mathbf{W} are related to Z^{abcd} , the variation of the Lagrangian with respect to the Riemann tensor R_{abcd} ,

$$Z^{abcd} = \frac{\delta^{cov} L}{\delta R_{abcd}}, \quad (3.8)$$

by,

$$(\mathbf{W}^c)_{c_3 c_4 c_5} = -2 \nabla_d Z^{abcd} \epsilon_{abc_3 c_4 c_5} = 2 (\nabla_d \mathbf{X}^{cd})_{c_3 c_4 c_5}. \quad (3.9)$$

The process of finding the central charge for the supersymmetric black ring follows the same recipe as for the Kerr solution. After some work one finds that the central charge associated to the Virasoro algebra (3.3) is,

$$c^{(k)} = -12e_0 \int_{\Sigma} Z_{abcd} \epsilon^{ab} \epsilon^{cd} \text{vol}(\Sigma) = \frac{6e_0}{\pi} S_{\text{mac}}. \quad (3.10)$$

In the last step we used the Iyer-Wald formula for macroscopic entropy of a black hole which is generalization of Bekenstein-Hawking formula when the higher derivative terms are appeared. So the central charge is

$$c^{(k)} = C_{IJK} p^I p^J p^K + c_{2I} p^I. \quad (3.11)$$

As we shall see in the next section, this central charge is equal to the left central charge computed by the c-extremization formalism. This equality was shown for black rings without higher derivative corrections in [8]. Finding this relation for the case with higher derivative terms is a much stronger result and unlikely to be a coincidence. We consider this equality further in the discussion section.

3.2 Microscopic entropy

The microscopic entropy of supersymmetric black ring in the Kerr/CFT approach can be computed by the following form of the Cardy formula,

$$S_{\text{mic}}^{(k)} = \frac{\pi^2}{3} c^{(k)} T_{FT}, \quad (3.12)$$

where T_{FT} is the Frolov-Thorne temperature. The Frolov-Thorne temperature is an intrinsic feature of metric and its definition is not corrected by higher derivative terms.⁶ So as usual, one can find the Frolov-Thorne temperature from the $t\psi$ cross term of near horizon geometry (3.4)⁷

$$T_{FT} = \frac{1}{\pi e_0}. \quad (3.13)$$

Using (3.12,3.13) one finds that

$$S_{\text{mic}}^{(k)} = \frac{2\pi}{e_0} \left(\frac{1}{6} C_{IJK} p^I p^J p^K + \frac{1}{6} c_{2I} p^I \right). \quad (3.14)$$

As we expect this microscopic entropy associated with the asymptotic symmetry group is equal to the macroscopic entropy (2.19).

4 C-extremization approach

One can also use the usual Cardy formula to compute the microscopic entropy of the supersymmetric black ring. The low energy decoupling limit implies that only the left-handed excitations survive and the microscopic entropy is given by,

$$S_{\text{mic}} = 2\pi \sqrt{\frac{c_L \hat{q}_0}{6}}. \quad (4.1)$$

⁶Appendix B of [17] is devoted to this subject.

⁷Often there is a factor of 2 in the denominator of the expression for the Frolov-Thorne temperature but not in our case since we have take the period of ψ to be 4π .

In this form of the Cardy formula c_L is the left central charge and \hat{q}_0 , given in eq.(2.18), corresponds to the left-handed excitations of the CFT. Using the c-extremization formalism one can compute this central charge from near horizon data. Although this formalism was introduced for a geometry with a globally AdS_3 part, we assume that it can also be used for geometries which are locally AdS_3 . We don't prove this but the self-dual orbifold AdS_3 perspective [26, 27] and EVH/CFT proposal [38] suggest that this approach can also be used for a locally AdS_3 geometry. A posteriori the fact that we obtain non-trivial agreement with the results of the previous section gives further weight to our assumption. We will discuss this point in last section.

At the leading order it was shown that the c-extremization and Brown-Henneaux (or Kerr/CFT) approach lead to the same result for the central charge [8]. At this level the left and right central charges are equal and given by

$$c_L = c_R = C_{IJK} p^I p^J p^K. \quad (4.2)$$

Turning on the higher order correction one can use the c-extremization approach to find the average of left and right central charges. Then, finding the gravitational anomaly gives the difference between left and right central charges so that combining the two one can obtain c_L and c_R .

The first step in applying the c-extremization formalism is choosing an appropriate ansatz,

$$ds^2 = l_{\text{AdS}_3}^2 ds_{\text{AdS}_3}^2 + l_{S^2}^2 ds_{S^2}^2, \quad (4.3)$$

$$A^I = e^I r dt - \frac{p^I}{2} \cos \theta d\phi + a^I (e_0 r dt + d\psi), \quad (4.4)$$

Then by extremizing the c-function,

$$c = 6 l_{\text{AdS}_3}^3 l_{S^2}^2 (\mathcal{L}_0 + \mathcal{L}_1), \quad (4.5)$$

with respect to, l_{AdS_3} and l_{S^2} , the AdS and sphere radii respectively, we find their values in terms of the magnetic charges. The value of c-function at these radii gives the average of left and right central charges. Performing these calculation one finds,

$$l_{\text{AdS}_3} = 2 l_{S^2} = \left(\frac{1}{6} C_{IJK} p^I p^J p^K + \frac{1}{12} c_{2I} p^I \right)^{1/3}, \quad (4.6)$$

$$A^I = e^I r dt - \frac{p^I}{2} \cos \theta d\phi + a^I (e_0 r dt + d\psi), \quad X^I = \frac{p^I}{l_{\text{AdS}^2}}, \quad D = \frac{12}{l_{\text{AdS}^2}^2}, \quad (4.7)$$

$$Q^I = -4 C_{IJK} p^J a^K, \quad e^I + e_0 a^I = 0, \quad v_{\theta\phi} = \frac{3}{8} l_{\text{AdS}^2} \sin \theta,$$

and the value of c-function at this extremum point is given by

$$c|_{\text{ext.}} = \frac{1}{2} (c_L + c_R) = C_{IJK} p^I p^J p^K + \frac{3}{4} c_{2I} p^I. \quad (4.8)$$

There is a precise agreement between the above solution and the results of entropy function formalism reviewed in the previous section (2.11).

In [28] it was shown that for the associated dual CFT the gravitational anomaly yields the difference between left and right central charges,

$$c_L - c_R = \frac{1}{2}c_{2I}p^I. \quad (4.9)$$

Thus the left and right central charges are given by

$$c_L = C_{IJK}p^I p^J p^K + c_{2I}p^I, \quad c_R = C_{IJK}p^I p^J p^K + \frac{1}{2}c_{2I}p^I, \quad (4.10)$$

Now we can use the Cardy formula (4.1) and equations (2.18) and (2.16) to compute the microscopic entropy of black ring,

$$S_{\text{mic}}^{(c)} = 2\pi\sqrt{\frac{c_L \hat{q}_0}{6}} = \frac{2\pi}{e_0} \left(\frac{1}{6}C_{IJK}p^I p^J p^K + \frac{1}{6}c_{2I}p^I \right). \quad (4.11)$$

The above entropy is in precise agreement with result of Kerr/CFT approach (3.14).

5 Summary and discussion

In this paper we study the microscopic interpretation of SUSY black ring solutions of $\mathcal{N} = 2$ supergravity in the presence of supersymmetric completion of mixed gauge-gravitational Chern-Simons term (2.3). Because of the near horizon geometry of these solutions, one can use both c-extremization and the Kerr/CFT approach to find the microscopic entropy via computing the associated central charge. We showed that central charge, which counts the degeneracy of ground states in the CFT side, is given by the magnetic charges of SUSY black rings (4.10) or (3.11) in both methods independently.

We found that the usual form of the Cardy formula (4.1) without any subleading corrections can be used for SUSY black ring solution even in the presence of higher derivative corrections (2.3).⁸ This works because the effect of the higher derivative corrections is relatively simple – essentially we just have a shift of the Kaluza-Klein photon charge (2.16, 2.18). As long as we remain in the large charge regime, we do not need to consider subleading corrections. It means our results are in agreement with the canonical ensemble description of black ring used in [40]. This discussion also applies to the use of (3.12).

The most interesting result of our study is that the Kerr/CFT and c-extremization approaches which are apparently related to different Virasoro algebras lead to the same value of central charges and the same microscopic entropies in a highly non-trivial setting.

In order compare these approaches, and to try get a handle on the various central charges appearing in the game, it is helpful to consider the geometry in detail. While global AdS_3 has two $SL(2, \mathbb{R})$ symmetries, as discussed in section 2, our near horizon only has one with the other $SL(2, \mathbb{R})$ is broken to a $U(1)$. Now, it would seem to be natural to associate c_L with the unbroken $SL(2, \mathbb{R})$ and conclude that the right-handed excitations are killed by the broken $SL(2, \mathbb{R})$. This however seems to be incompatible with the fact that it is precisely the residual part of the broken $SL(2, \mathbb{R})$, \bar{L}_0 which forms the basis of the Virasoro algebra associated with the asymptotic symmetry group considered in the Kerr/CFT approach.

⁸This was previously shown for BTZ black hole solutions of 3D gravity [39].

The incompatibility is however not so stark once one realises that c-extremization focuses on the near horizon geometry of the solutions, while the Kerr/CFT approach is based on the fluctuations at the boundary (of the near horizon geometry). Our results are strong evidence that both approaches count microstates of the same CFT but in different ways. In fact, the generators of the unbroken $SL(2, \mathbb{R})$, $L_0, L_{\pm 1}$, do not commute with the Virasoro generators of the Kerr/CFT approach (3.2) which means they are not independent. This suggests that although the asymptotic symmetry group Virasoro algebra is “based” on the $U(1)$ part of the near horizon isometry, the $SL(2, \mathbb{R})$ plays a crucial role in Kerr/CFT correspondence.

This leads us to conclude that both points of view are somehow talking to each other. The central charge counts the ground states dual to SUSY black rings and the number of these states is independent of any approach used to this counting.

This is in agreement with the DLCQ approach which relates the left-handed excitations of the self-dual orbifold AdS_3 geometry on two distinct boundaries [41, 42, 43]. In our case, The S^1 part, which carries the angular momentum of the black ring, is fibred over the AdS_2 part such that locally one has an AdS_3 geometry called a self-dual orbifold of AdS_3 [26, 27]. From this viewpoint it was shown that extremal BTZ black hole, which has self-dual orbifold AdS_3 near horizon geometry, is dual to discrete light cone quantised CFT_2 which admits one chiral Virasoro algebra [41, 42, 43].

Our results suggest some further possible avenues to investigate. It would be interesting to compare three dimensional extremal BTZ black holes in the presence of some higher derivative terms and light cone quantised CFT_2 . Another interesting avenue to consider is whether hidden conformal symmetries appear beyond the extremal limit for SUSY black rings with higher derivative corrections. In this situation one expects both left and right central charges to be excited.

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